Joint Gaussian Graphical Model Series – VIII

A deep introduction of the metrics for evaluating an/a estimator/learner

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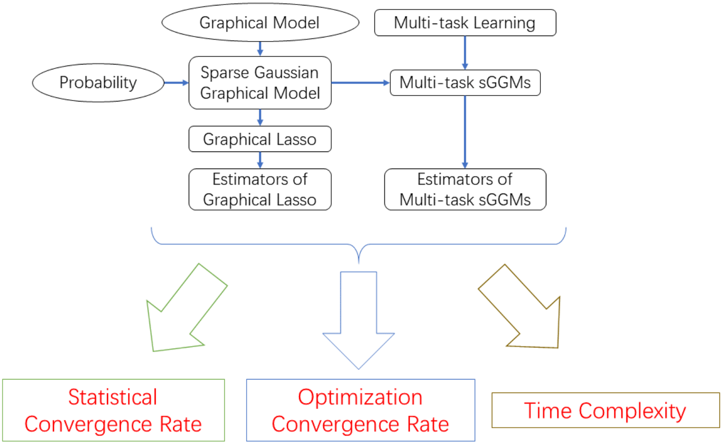
Road Map


Graphical Model
Multi-task Learning
Probability
Sparse Gaussian Graphical Model
Multi-task sGGMs
Graphical Lasso
Estimators of Graphical Lasso
Estimators of Multi-task sGGMs
Statistical Convergence Rate
Optimization Convergence Rate
Time Complexity

Outline

Notation

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- X The data matrix
- Σ The covariance matrix.
- Ω The precision matrix.
- p The number of features.
- n The number of samples in the data matrix.
- s The number of non-zero entries in the precision matrix.

Review

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- We introduce different multi-task sGGMs estimators and their optimization challenges.

The metrics for evaluating an estimator

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- You need some metrics to make the decision.

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- You want to know whether this novel estimator is no worse than the previous ones.
- Then you need some metrics to evaluate the estimator.

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- Accuracy:
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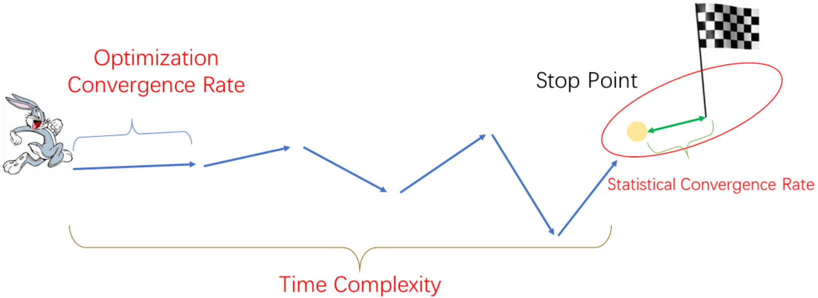
- Speed:
  - Optimization convergence rate
  - Optimization researchers
  - Computational complexity
  - Computer Scientists

Overview Figure



Statistical Convergence Rate

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- Suppose the parameter you need to estimate is θ, the truth is θ^*
- $\| \theta - \theta^* \|$ or $\mathcal{R}(\theta - \theta^*)$

A simple example: Estimate the mean

On the whiteboard.

Elementary Estimator[Yang et al.(2014b)Yang, Lozano, and Ravikumar]

$$\underset{\theta}{\text{argmin}} \mathcal{R}(\theta) \tag{4.1}$$

$$\text{Subject to: } \mathcal{R}^*(\theta - \mathcal{B}^*(\widehat{\phi})) \leq \lambda_n \tag{4.2}$$

Here $\mathcal{B}^*(\widehat{\phi})$ is a backward mapping for $\widehat{\phi}$.

Example: sparse linear regression[Yang et al.(2014a)Yang, Lozano, and Ravikumar]

$$\underset{\theta}{\operatorname{argmin}} ||\theta||_1 \tag{4.3}$$

$$\text{Subject to: } ||\theta - (X^T X + \epsilon I)^{-1} X^T y||_\infty \leq \lambda_n \tag{4.4}$$

Hands on: Elementary Estimator for high-dimensional linear regression

On the whiteboard.

Hands on: DIFFEE

On the whiteboard.

Conclusions

- In high-dimensional setting, related to $\frac{\log p}{n}$.
- Equivalent estimators still have differences in constants or constraints

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- Closed form solution
- Closed form $\geq$ Higher order $\geq$ linear

Optimization Convergence Rate: Basic Results

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- Newton method based method: Quadratic
- Elementary Estimator: Closed form solution

Optimization Convergence Rate: Different methods

	Single sGGM			Multiple sGGMs	
Method:	GLasso	CLIME	EEGM	JGL	FASJEM
Rate of Convergence	Linear	NA	Closed form	Linear	Linear

Computational Complexity

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- In machine learning, it is mainly related to n and p.
- Use big O notation

Computational Complexity: how to calculate

- Some cases:
  - Matrix Multiplication: $O(np^2)$
  - Matrix inversion $O(p^3)$
  - SVD inversion $O(p^3)$
  - soft-thresholding $O(p^2)$

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- Some cases:
  - Matrix Multiplication: $O(np^2)$
  - Matrix inversion $O(p^3)$
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  - soft-thresholding $O(p^2)$
- How to calculate:
  - Num of Iter $\times$ Computational complexity of each Iter
  - Direct calculate e.g., Closed form solution
  - Use existing method e.g., linear programming
  - Special case: linear convergence.

Computational Complexity: Different methods

	Single sGGM			Multiple sGGMs		
Method:	GLasso	CLIME	EEGM	JGL	FASJEM	SIMUL
Computational Complexity	$O(Tp^2)$	$O(p^5)$	$O(p^2)$	$O(Tp^3)$	$O(Tp^2)$	$O(K^4 p$

Summary

- We introduce the statistical convergence rate.
- We introduce the optimization convergence rate.
- We introduce the computational complexity.

References I

E. Yang, A. Lozano, and P. Ravikumar.
Elementary estimators for high-dimensional linear regression.
In *Proceedings of the 31st International Conference on Machine Learning (ICML-14)*, pages 388–396, 2014a.

E. Yang, A. C. Lozano, and P. K. Ravikumar.
Elementary estimators for graphical models.
In *Advances in Neural Information Processing Systems*, pages 2159–2167, 2014b.