Spectral Networks and Deep Locally Connected Networks on Graphs

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1 Introduction

CNNs have been very successful with problems where the coordinates of the underlying data structure have a grid structure, and the data to be studied has translational equivariance w.r.t. the grid.

Problem: Not all data is like this; for example, 3-D mesh data like surface tension and temperature or data from social networks.

Approaches:

- Spatial construction: Extend compact filters and multiscale dyadic clustering of grids to graphs to define "locally" connected and pooling layers which require $O(n)$ instead of $O(n^2)$ parameters
- Spectral construction: Draw on properties of convolutions in the Fourier domain

For typical CNN: n input coordinates on a grid of d dimensions, fully connected layer with m outputs; results in $O(mn) \approx O(n^2)$ parameters.

2 Spatial Construction

Replace grid with weighted graph $G = (\Omega, W)$, where Ω is a discrete set of size m and $W \in R^{mxm}$ is symmetric and nonnegative.

Define neighborhoods on W: $N_\delta(j) = \{i \in \Omega : W_{ij} < \delta\}$. Restrict attention to sparse "filters" with receptive fields given by these neighborhoods to restrict parameters to $O(Sn)$, where S is average neighborhood size.

Grids naturally have multiscale clustering. On grids, dyadic clustering behaves nicely w.r.t. the metric and the Laplacian: use naive agglomerative method to find multiscale clusterings in graph.

FIGURE

complicated formulas :(

3 Spectral Construction

4 Numerical Experiments

Two datasets: subsampled MNIST (to remove grid structure) and MNIST projected onto sphere

5 Conclusion

- Use graph-based analogies of convolutional architectures can greatly reduce the number of parameters without worsening (and often improving) the test error and increase the speed of forward propagation
- Possibly with more careful training and deeper networks, results can be improved on manifold-like graphs
- Intend to apply models to less artificial problems (like Netflix recommendations)