Three Deep Generative Models

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https://qdata.github.io/deep2Read/

Generalized Denoising Auto-Encoders as Generative Models

(Bengio et. al. - NIPS 2013)

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Question: Do denoising auto-encoders completely characterize the input distribution or only some aspect of it?

- Clustering algorithms only capture the modes of the distribution, while manifold learning algorithms characterize the low-dimensional regions where the density concentrates

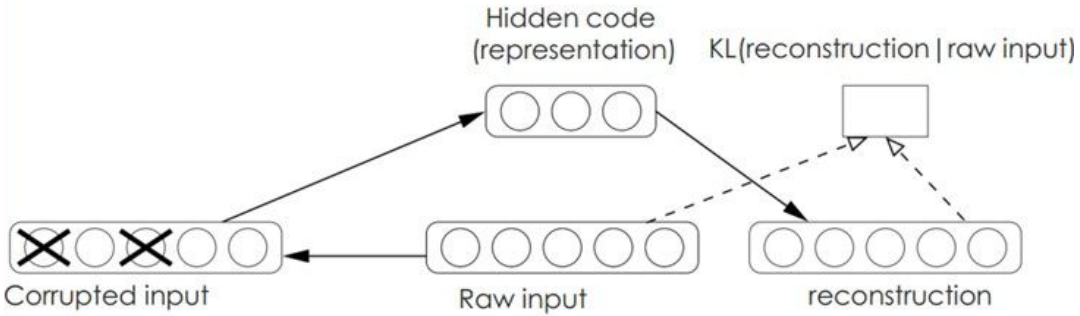
Propose: Different probabilistic interpretation of DAEs, which is valid for any data type

Generalized Denoising Auto-Encoders as Generative Models

(Bengio et. al. - NIPS 2013)

Overview:

The basic idea is that if we corrupt observed random variable X into $\tilde{X}$ using conditional distribution $C(\tilde{X}|X)$, we are really training the DAE to estimate the reverse conditional $P(X|\tilde{X})$. Combining this estimator with the known $C(\tilde{X}|X)$, we show that we can recover a consistent estimator of $P(X)$ through a Markov chain that alternates between sampling from $P(X|\tilde{X})$ and sampling from $C(\tilde{X}|X)$, i.e., encode/decode, sample from the reconstruction distribution model $P(X|\tilde{X})$, apply the stochastic corruption procedure $C(\tilde{X}|X)$, and iterate.



Generalized Denoising Auto-Encoders as Generative Models

(Bengio et. al. - NIPS 2013)

Training:

Algorithm 1 THE GENERALIZED DENOISING AUTO-ENCODER TRAINING ALGORITHM *requires a training set or training distribution $\mathcal{D}$ of examples X, a given corruption process $C(\tilde{X}|X)$ from which one can sample, and with which one trains a conditional distribution $P_\theta(X|\tilde{X})$ from which one can sample.*

repeat

- sample training example $X \sim \mathcal{D}$
- sample corrupted input $\tilde{X} \sim C(\tilde{X}|X)$
- use $(X, \tilde{X})$ as an additional training example towards minimizing the expected value of $-\log P_\theta(X|\tilde{X})$, e.g., by a gradient step with respect to θ.

until convergence of training (e.g., as measured by early stopping on out-of-sample negative log-likelihood)

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Sampling:

We define the following pseudo-Gibbs Markov chain associated with P_θ:

$$X_t \sim P_\theta(X|\tilde{X}_{t-1})$$
$$\tilde{X}_t \sim \mathcal{C}(\tilde{X}|X_t) \tag{3}$$

which can be initialized from an arbitrary choice X_0. This is the process by which we are going to generate samples X_t according to the model implicitly learned by choosing θ. We define $T(X_t|X_{t-1})$ the transition operator that defines a conditional distribution for X_t given X_{t-1}, independently of t, so that the sequence of X_t's forms a homogeneous Markov chain. If the asymptotic marginal distribution of the X_t's exists, we call this distribution $\pi(X)$, and we show below that it consistently estimates $\mathcal{P}(X)$.

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Figure 4: *Successive samples generated by Markov chain associated with the trained DAEs according to the plain sampling scheme (left) and walkback sampling scheme (right). There are less "spurious" samples with the walkback algorithm.*

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Conclusion:

Training a model to denoise is a way to implicitly estimate the underlying data generating process, and that a *simple Markov chain that alternates sampling from the denoising model and from the corruption process converges to that estimator.* This provides a means for generating data from any DAE

The Neural Autoregressive Distribution Estimator

(Larochelle and Murray - AISTATS 2011)

The Neural Autoregressive Distribution Estimator

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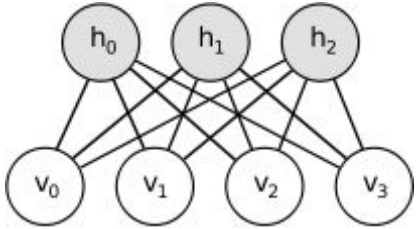
RBMs model the *distribution of observations* using binary hidden variables

$$E(\mathbf{v}, \mathbf{h}) = -\mathbf{h}^\top \mathbf{W}\mathbf{v} - \mathbf{b}^\top \mathbf{v} - \mathbf{c}^\top \mathbf{h} \qquad (1)$$

probabilities are assigned to any observation $\mathbf{v}$ as follows:

$$p(\mathbf{v}) = \sum_{\mathbf{h}} \exp(-E(\mathbf{v}, \mathbf{h}))/Z\,, \qquad (2)$$



RBM Inference: If the observations can be decomposed into an input x and a target y, then an RBM trained on such pairs can also be used to predict a new x

The Neural Autoregressive Distribution Estimator

(Larochelle and Murray - AISTATS 2011)

Problem: RBM isn't suited for estimating the joint probability of a given observation (due to partition function being intractable)

Solution: Convert RBM into Bayesian Network

$$p(\mathbf{v}) = \prod_{i=1}^{D} p(v_i | \mathbf{v}_{\text{parents}(i)}) \,,$$

where all observation variables v_i are arranged into a directed acyclic graph and $\mathbf{v}_{\text{parents}(i)}$ corresponds to all the variables in $\mathbf{v}$ that are parents of v_i into that graph.

The Neural Autoregressive Distribution Estimator

(Larochelle and Murray - AISTATS 2011)

$$p(\mathbf{v}) = \prod_{i=1}^{D} p(v_i|\mathbf{v}_{<i})$$

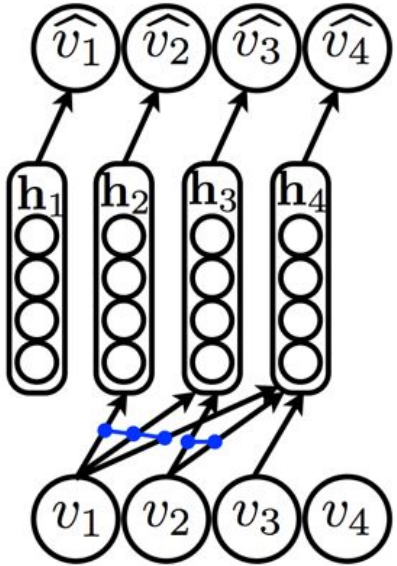
$$= \prod_{i=1}^{D} p(v_i, \mathbf{v}_{<i})/p(\mathbf{v}_{<i})$$

$$= \prod_{i=1}^{D} \frac{\sum_{\mathbf{v}_{>i}} \sum_{\mathbf{h}} \exp(-E(\mathbf{v}, \mathbf{h}))}{\sum_{\mathbf{v}_{j\geq i}} \sum_{\mathbf{h}} \exp(-E(\mathbf{v}, \mathbf{h}))} \,, \longrightarrow \text{Intractable} \longrightarrow \text{Approximate}$$

The Neural Autoregressive Distribution Estimator

(Larochelle and Murray - AISTATS 2011)

$$p(v_i = 1|\mathbf{v}_{<i}) = \text{sigm}\left(b_i + (\mathbf{W}^\top)_{i,.}\mathbf{h}_i\right)$$
$$\mathbf{h}_i = \text{sigm}\left(\mathbf{c} + \mathbf{W}_{.,<i}\mathbf{v}_{<i}\right),$$



NADE

Training is done by minimizing the average negative log-likelihood of the parameters given the training set:

$$\frac{1}{T}\sum_{t=1}^{T} -\log p(\mathbf{v}_t) = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{D} -\log p(v_i|\mathbf{v}_{<i}), \quad (11)$$

The Neural Autoregressive Distribution Estimator

(Larochelle and Murray - AISTATS 2011)

Conclusion:

NADE can be seen as a method for converting an RBM into a tractable distribution estimator

It can also be understood as a special kind of autoencoder whose output assigns valid probabilities to observations and hence is a proper generative model

Generative Adversarial Networks

(Goodfellow et. al. - NIPS 2014)

Generative Adversarial Networks

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Overview:

Framework for estimating generative models via an adversarial process

Simultaneously train two models:

(1) generative model *G* that captures the data distribution,

(2) discriminative model D that estimates the probability that a sample came from the training data rather than G

Generative Adversarial Networks

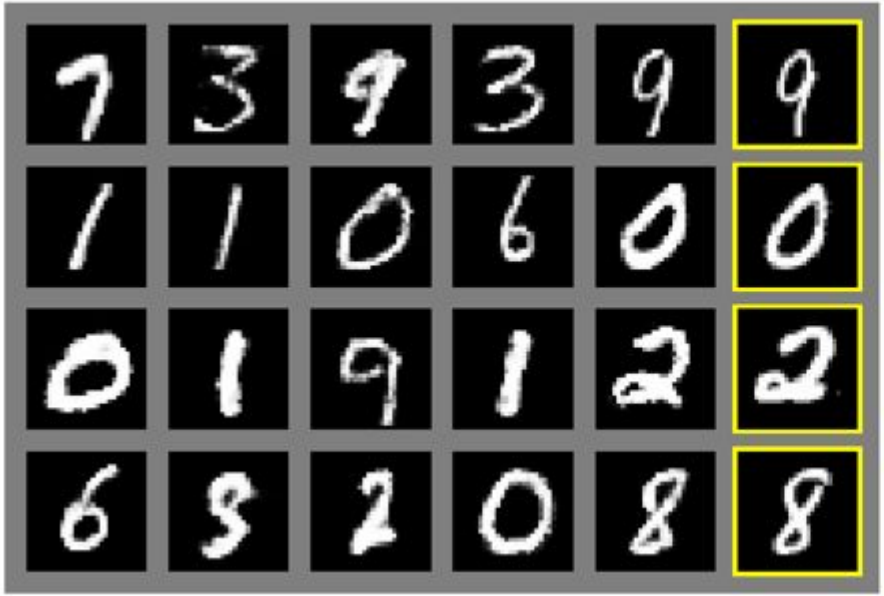
(Goodfellow et. al. - NIPS 2014)

Method:

The adversarial modeling framework is most straightforward to apply when the models are both multilayer perceptrons. To learn the generator's distribution p_g over data $\boldsymbol{x}$, we define a prior on input noise variables $p_{\boldsymbol{z}}(\boldsymbol{z})$, then represent a mapping to data space as $G(\boldsymbol{z}; \theta_g)$, where G is a differentiable function represented by a multilayer perceptron with parameters θ_g. We also define a second multilayer perceptron $D(\boldsymbol{x}; \theta_d)$ that outputs a single scalar. $D(\boldsymbol{x})$ represents the probability that $\boldsymbol{x}$ came from the data rather than p_g. We train D to maximize the probability of assigning the correct label to both training examples and samples from G. We simultaneously train G to minimize $\log(1 - D(G(\boldsymbol{z})))$:

Generative Adversarial Networks

(Goodfellow et. al. - NIPS 2014)



Unlike most other visualizations of deep generative models, these images show actual samples from the model distributions, not conditional means given samples of hidden units. Moreover, these samples are uncorrelated because the sampling process does not depend on Markov chain mixing

Summary of Prev 3 Models

	Deep directed graphical models	Deep undirected graphical models	Generative autoencoders	Adversarial models
Training	Inference needed during training.	Inference needed during training. MCMC needed to approximate partition function gradient.	Enforced tradeoff between mixing and power of reconstruction generation	Synchronizing the discriminator with the generator. Helvetica.
Inference	Learned approximate inference	Variational inference	MCMC-based inference	Learned approximate inference
Sampling	No difficulties	Requires Markov chain	Requires Markov chain	No difficulties
Evaluating $p(x)$	Intractable, may be approximated with AIS	Intractable, may be approximated with AIS	Not explicitly represented, may be approximated with Parzen density estimation	Not explicitly represented, may be approximated with Parzen density estimation
Model design	Nearly all models incur extreme difficulty	Careful design needed to ensure multiple properties	Any differentiable function is theoretically permitted	Any differentiable function is theoretically permitted