Summary of Basics about Generative Adversarial Network

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https://qdata.github.io/deep2Read/

Generative model

- Training and sampling from generative model shows how our model can represent high-dimensional probability distribution
- Generative model can be used on reinforcement learning
- Generative model can perform semi-supervised learning
- Generative model can enable machine learning to work with multi-modal output
- Many tasks intrinsically requires the generation of good samples

Maximum Likelihood Estimation

- Maximum likelihood:
  - Choose the parameters to maximize the likelihood to training data
  - Easier to do in the log space

$$\theta^* = \arg\max_{\theta} \Pi_{i=1}^{\mathrm{m}} p_{model}(x^{(i)};\theta\ )$$

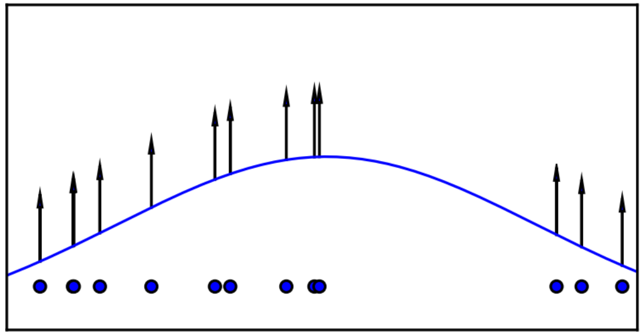
$$= \arg\max_{\theta} \sum_{i=1}^{m} \log p_{model}(x^{(i)};\theta)$$

$$= \arg\max_{\theta} \int \mathrm{p_{data}}(\mathrm{x}) \log p_{model}(x;\theta)\, dx$$

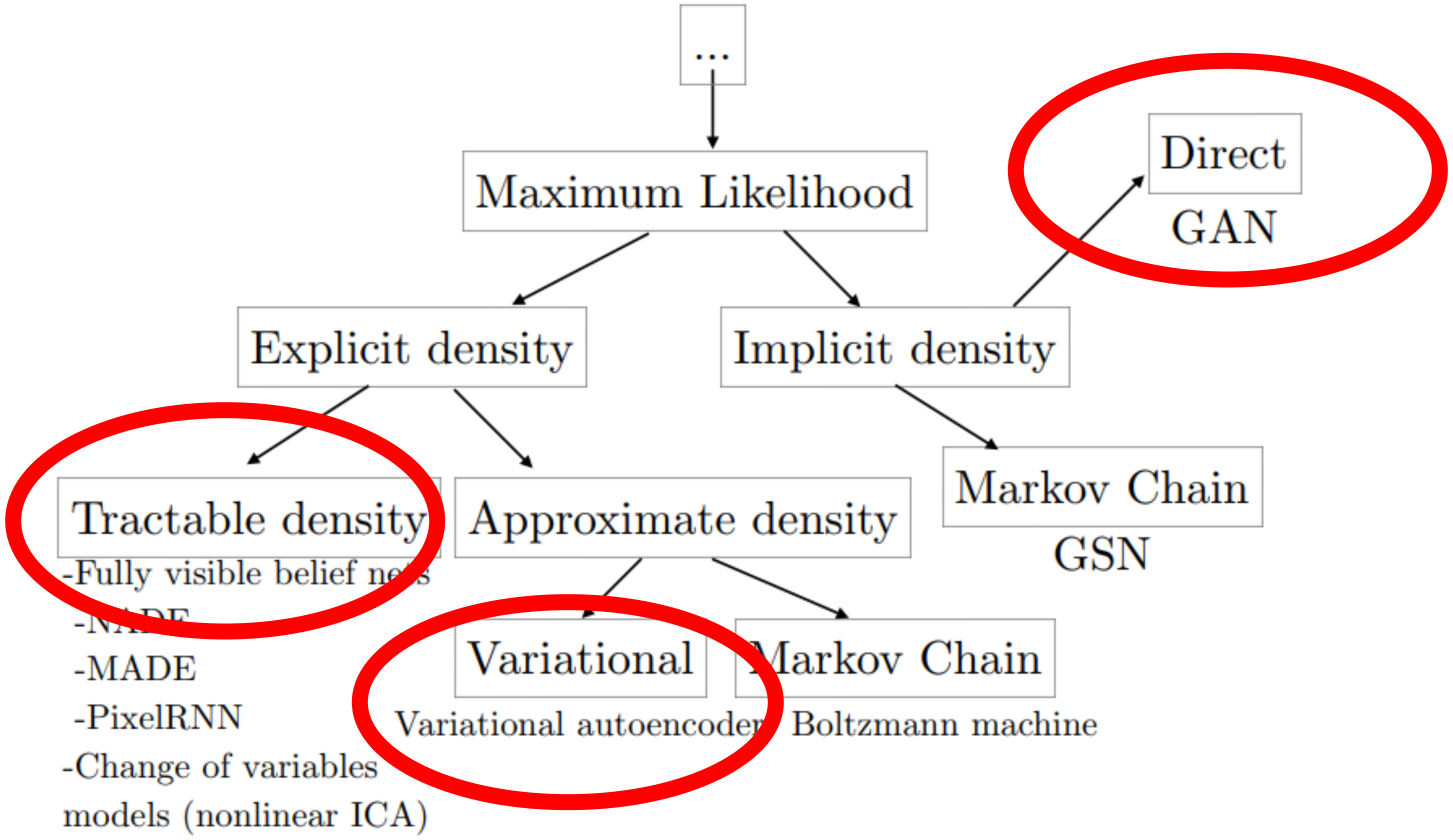
$$= \arg\max_{\theta} \int \mathrm{p_{data}}(\mathrm{x}) \log p_{model}(x;\theta) - p_{data}(x) \log p_{data}(x)\, dx$$

$$= \arg\min_{\theta} \int \mathrm{p_{data}}(\mathrm{x}) \log \frac{p_{data}(x)}{p_{model}(x)} dx$$

$$= \arg\min_{\theta} D_{KL}(p_{data} || p_{model}(\theta))$$ KL divergence!



Generative model Tree (NIPS 2016 Tutorial)



Autoregressive models

$$p_{model}(x;\theta) = \prod_{d=1}^{D} p(x_d | x_{1:d-1};\theta)$$

Deep Belief Network

NADE

MADE

PixelRNN

PixelCNN++

WaveNet

- Define explicit density function
- Using chain rule

Variational Autoencoder

- Maximize variational lower bound $L(x;\theta) \leq \log p_{model}(x;\theta)$
- Guarantee to achieve a high value as the log-likelihood
- Issue:
  - Bias: If not properly defined, the gap might be an issue
  - Tend to get good likelihood, but bad quality on image

Generative Adversarial Network(GAN)

- **Generator:** Some differentiable function G(z). Z is sampled from some simple prior distribution, G(z) is a sample of x drawn from p_{model}.
  - Structure can be anything
- **Discriminator:** Traditional supervise learning model

Math: Cost Function of GAN

- **Discrimintor:** $J^{(D)}\left(\theta^{(D)}, \theta^{(G)}\right) = -E_{x\sim p_{data}} \log D(x) - E_Z \log(1 - D(G(z)))$
- Cross entropy loss. Label 1 for the samples in the dataset, 0 for generator samples.
- Discriminator is able to estimate $p_{data}(x)/p_{model}(x)$ everywhere
- It doesn't have any bias.
- **Generator:** $J^{(G)}\left(\theta^{(D)}, \theta^{(G)}\right) = -J^{(D)}(\theta^{(D)}, \theta^{(G)})$ **if it's a zero-sum game**
- In zero-sum game case the game resembles Jenson-Shannon Bound.

Jenson-Shannon Bound

- Suppose D is optimal, we have $\mathrm{D}(x;\theta_D^*) = \frac{p_{data}(x)}{p_{data}(x)+p_{model}(x)}$
- In this case $J(\theta_D^*,\theta_G) = E_{x\sim p_d}\log D(x) + E_z \log\left(1 - D\left(G(z)\right)\right)$
- $= \int p_{data}(x)\frac{p_{data}(x)}{p_{data}(x)+p_{model}(x)}dx + \int p_g(x)\frac{p_{data}(x)}{p_{data}(x)+p_{model}(x)}dx$
- $= KL\left(p_{data}||\frac{p_{data}(x)+p_{model}(x)}{2}\right) + KL(p_{model}||\frac{p_{data}(x)+p_{model}(x)}{2}) - 2\log 2$
- $= 2\mathrm{Jenson{-}Shannon}(p_{data}||p_{model}) - 2\log 2$

Math: Cost Function of GAN II

- **Discrimintor:** $J^{(D)}\left(\theta^{(D)}, \theta^{(G)}\right) = -E_{x\sim p_{data}} \log D(x) - E_Z \log(1 - D(G(z)))$
- **Generator:** $J^{(G)}\left(\theta^{(D)}, \theta^{(G)}\right) = -J^{(D)}(\theta^{(D)}, \theta^{(G)})$ **if it's a zero-sum game**
- However, zero-sum game doesn't perform well in practice:
  - Generator maximize the cross entropy
  - If discriminator performs well, the gradient of generator vanishes
- Solution from Goodfellow 2014: Change the sign
- $J^{(G)}\left(\theta^{(D)}, \theta^{(G)}\right) = -E_Z \log(D(G(z)))$
- No longer zero-sum, and heuristically designed

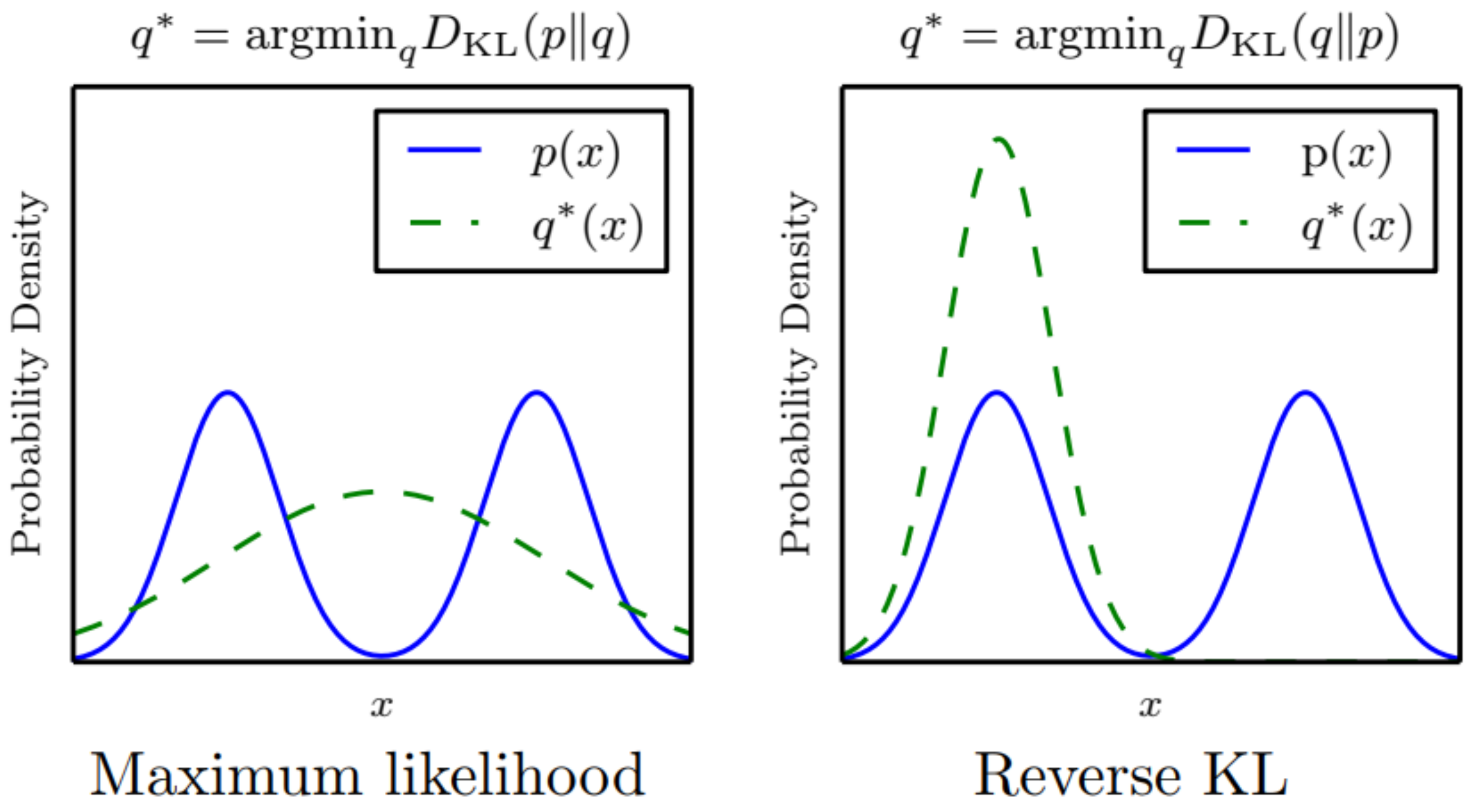
Math: Cost Function of GAN III

- If let $J^{(G)}\left(\theta^{(D)}, \theta^{(G)}\right) = -E_Z \sigma^{-1}\left(D\left(G(z)\right)\right)$:
- $J^{(G)}\left(\theta^{(D)}, \theta^{(G)}\right) = \int p_{model}(x) e^{\sigma^{-1}(D(G(z)))}\, dx$
- We have $e^{\sigma^{-1}(x)} = \frac{x}{1-x}$, therefore $e^{\sigma^{-1}(D(x)^*)} = \frac{p_{data}(x)}{p_{model}(x)}$
- $J^{(G)}\left(\theta^{(D)}, \theta^{(G)}\right) = -\int p_{model}(x) \log \frac{p_{data}(x)}{p_{model}(x)}\, dx = D_{KL}(p_{model} || p_{data})$

- By changing loss function from g, GAN can work on different loss

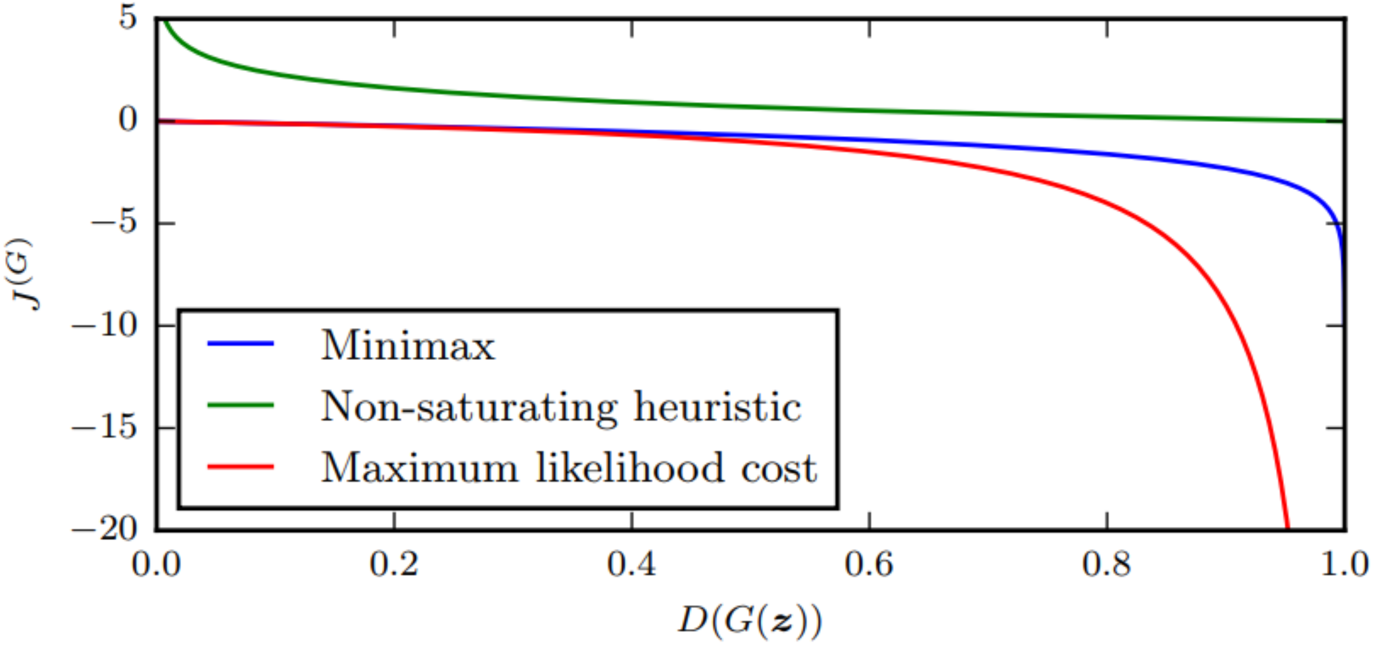
Choosing loss functions

- VAE use KL(MLE), whi
tends to get blurred
image
- Reverse KL on the oth
side, tend to converge
to modes
- GAN(original version)
optimize JS, somehow
similar to reverse KL
- However, it sometime
generate samples onl
from few modes



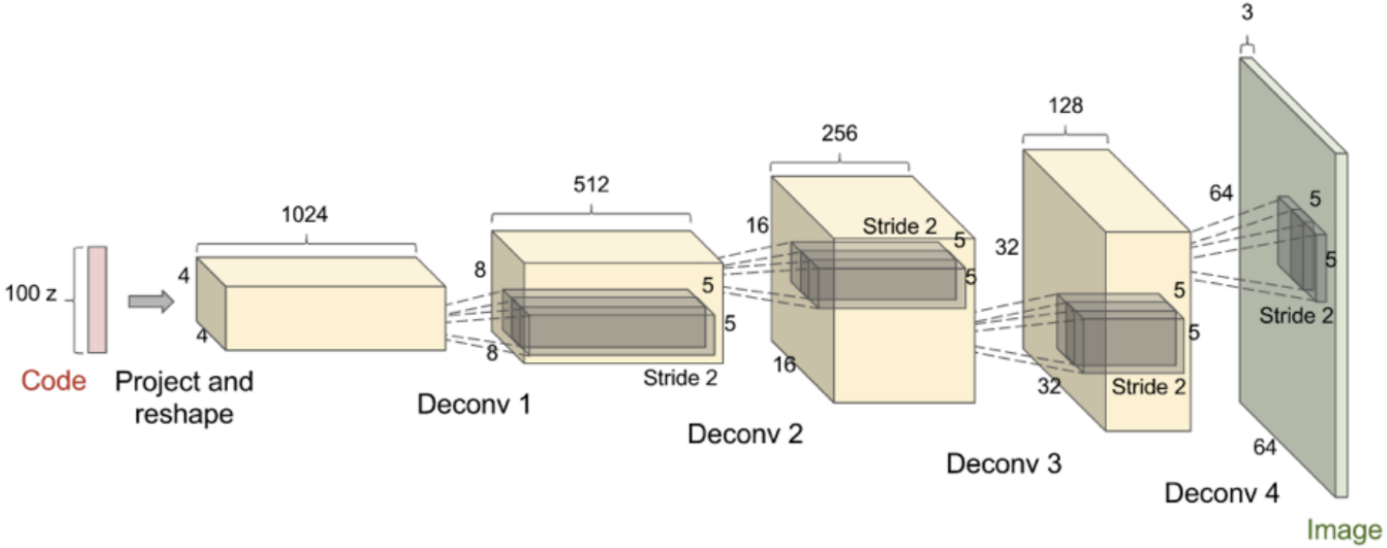
Choosing loss functions

- If the optimization is easy, minimax give no gradient
- The change is rapid on the right side, makes small number of samples dominating



Architecture: DCGAN

- Keys:
  - Use batch normalization, with the two minibatches for the discriminator normalized separately
  - All-convolutional net(Springenberg et al., 2015), no pooling
  - Adam instead of SGD



GAN tricks

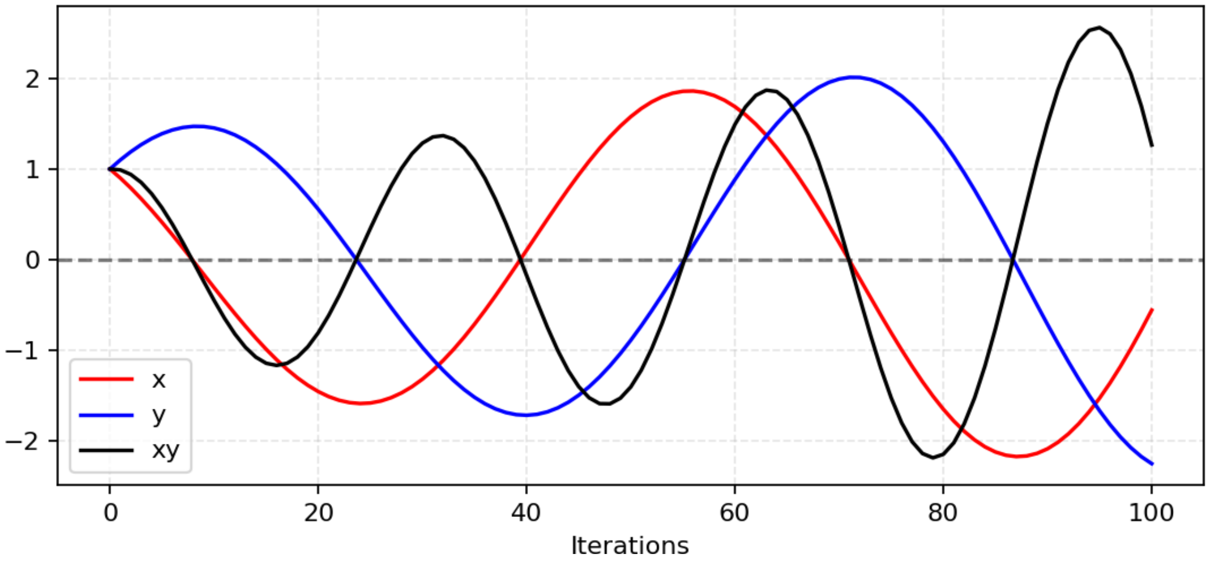
- https://github.com/soumith/ganhacks
- Train with labels can improve the performance drastically
- “One-sided” Label smoothing: Replace the target for the real examples with a value slightly less than 1, such as .9
- Batch normalization
- Adam

Convergence of GAN(Or Nonconvergence)

- For two player game, even if each player successfully moves downhill on that player's update, the same update might move the other player uphill.
- It converge on some game but not all.
- Mode collapse could be one result of such nonconvergence (rather than loss function). WGAN claims it alleviate the mode collapse problem, though.

Non-Convergence of GAN: Toy Example

- Toy example of game:
  - $V(x, y) = xy$
  - P1 Minimize V(x,y) by controlling y
  - P2 Maximize V(x,y) by controlling x
- Equilibrium: $x = y = 0$
- Gradient: $\Delta x = -\alpha \frac{\partial V}{\partial x} = -\alpha y$
- $\frac{\partial y}{\partial t} = \alpha \frac{\partial V}{\partial y} = \alpha x$



Convergence proof

- If g is on all functions, p_{model} converge to p_{data}
- However, it's not the case in the deep neural network

Proposition 2. *If G and D have enough capacity, and at each step of Algorithm 1, the discriminator is allowed to reach its optimum given G, and p_g is updated so as to improve the criterion*

$$\mathbb{E}_{\boldsymbol{x}\sim p_{data}}[\log D^*_G(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x}\sim p_g}[\log(1 - D^*_G(\boldsymbol{x}))]$$

then p_g converges to p_{data}

Proof. Consider $V(G, D) = U(p_g, D)$ as a function of p_g as done in the above criterion. Note that $U(p_g, D)$ is convex in p_g. The subderivatives of a supremum of convex functions include the derivative of the function at the point where the maximum is attained. In other words, if $f(x) = \sup_{\alpha\in\mathcal{A}} f_\alpha(x)$ and $f_\alpha(x)$ is convex in x for every α, then $\partial f_\beta(x) \in \partial f$ if $\beta = \arg\sup_{\alpha\in\mathcal{A}} f_\alpha(x)$. This is equivalent to computing a gradient descent update for p_g at the optimal D given the corresponding G. $\sup_D U(p_g, D)$ is convex in p_g with a unique global optima as proven in Thm 1, therefore with sufficiently small updates of p_g, p_g converges to p_x, concluding the proof. $\square$

Evaluation of generative models

- Hard to evaluate because doesn’t give probability on the training data
- Inception score?

Other scores:

- MODE score
- AM Score

Reference

- 1. Ian Goodfellow "NIPS 2016 tutorial: Generative adversarial networks." *arXiv preprint arXiv:1701.00160* (2016).
- 2. https://github.com/soumith/ganhacks