Towards Evaluating the Robustness of Neural Networks

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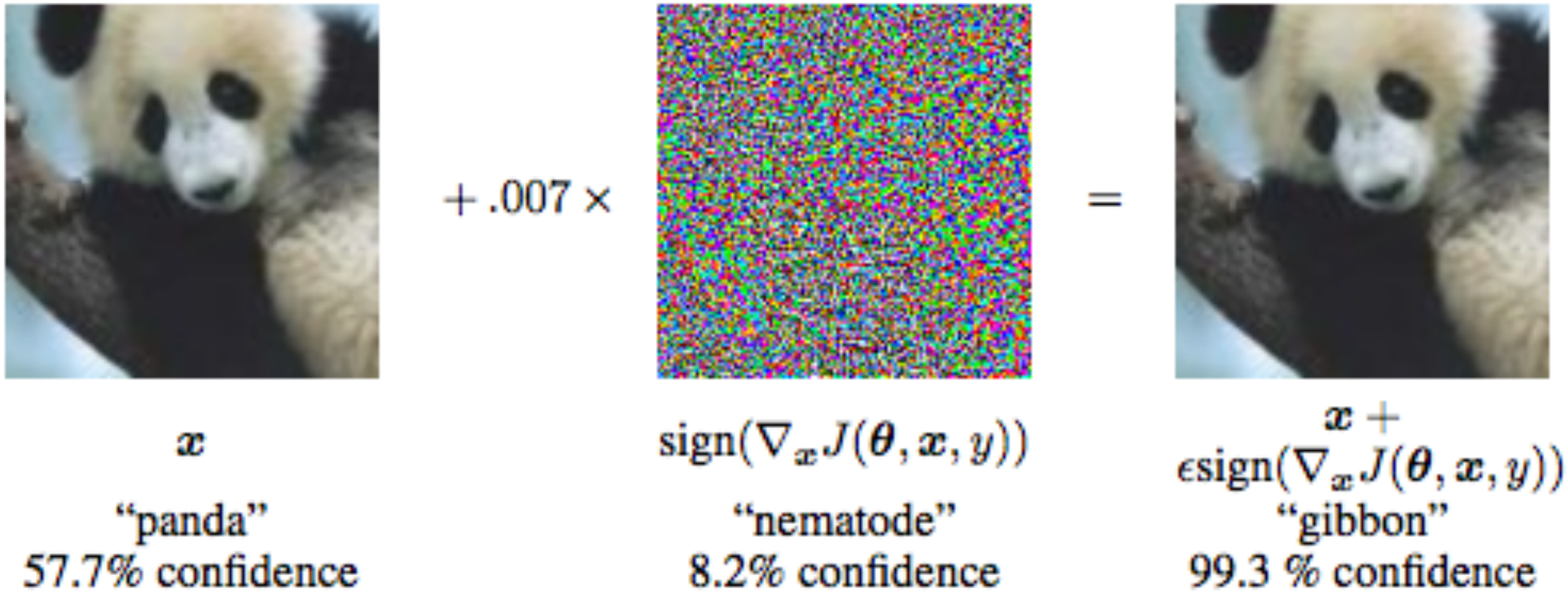
Adversarial Example



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Formalization

- Neural network as a function $g : X \to Y$,

$$g(\mathbf{x}) = f_L(f_{L-1}(\ldots((f_1(\mathbf{x})))))$$ (Typically softmax() as last layer.)

- The goal of an adversary in evasion attack
  - Given $\mathbf{x} \in X$ and $g(\cdot)$, find an $\mathbf{x}' \in X$ such that:

Untargeted: $$g(\mathbf{x}) \neq g(\mathbf{x}') \wedge \Delta(\mathbf{x}, \mathbf{x}') \leq \varepsilon$$

Or targeted: $$g(\mathbf{x}) = l \wedge \Delta(\mathbf{x}, \mathbf{x}') \leq \varepsilon$$

Distance Metric

- L^p-norm $\|x\|_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{\frac{1}{p}}$
  - L^2-norm
    $$\|x\|_2 = \left(x_1^2 + x_2^2 + \cdots + x_n^2\right)^{\frac{1}{2}}$$
  - L^∞-norm
    $$\|x\|_\infty = \max\{|x_1|, |x_2|, \ldots, |x_n|\}$$
  - L^0-“ norm”
    $$|x_1|^0 + |x_2|^0 + \cdots + |x_n|^0 \quad (0^0 = 0)$$
  - …

Which one is the best for vision task?

Existing Attacks

- L^2 Adversary
  - L-BFGS
- L^∞ Adversary
  - Fast Gradient Sign Method
  - Iterative Gradient Sign Method
- L^0 Adversary
  - Jacobian-based Saliency Map Approach

L^2 Adversary: L-BFGS

- Straightforward form, difficult to solve directly.

$$\begin{aligned} \text{minimize} \quad & \|x - x'\|_2^2 \\ \text{such that} \quad & C(x') = l \\ & x' \in [0,1]^n \end{aligned}$$

- Revised form

$$\begin{aligned} \text{minimize} \quad & c \cdot \|x - x'\|_2^2 + \text{loss}_{F,l}(x') \\ \text{such that} \quad & x' \in [0,1]^n \end{aligned}$$

  - Softmax-cross-entropy loss
  - Using L-BFGS-B as solver, which supports box constraints.
  - Try many values of c to get the minimum L^2

L^∞ Adversary: FGSM & iterative

- Fast Gradient Sign Method

$$x' = x - \epsilon \cdot \mathrm{sign}(\nabla \mathrm{loss}_{F,t}(x))$$

- Iterative Gradient Sign Method

$$x'_0 = x$$

$$x'_i = \mathrm{clip}_\epsilon(\, x'_{i-1} - \alpha \cdot \mathrm{sign}(\nabla \mathrm{loss}_{F,t}(x'_{i-1})))$$

L^0 Adversary: JSMA

- Jacobian-based Saliency Map Approach (JSMA)
- Basic idea: find the most influential pixels and change to maximum or minimum
- Iterative algorithm:
  1. If misclassified, terminate.
  2. Calculate the saliency map (Jacobian matrix).
  3. Pick a pair of pixels that will ①enlarge the score of target label and ②lower the score on other labels.
  4. Modify the pixel pair to maximum (or minimum) values. Goto 1.

Carlini's Attacks

- L^2 Adversary
- L^0 Adversary
- L^∞ Adversary

Carlini's L^2 Adversary

- Using logits-based objective instead of softmax-cross-entropy loss.

$$C(x+\delta) = t \text{ if and only if } f(x+\delta) \leq 0$$

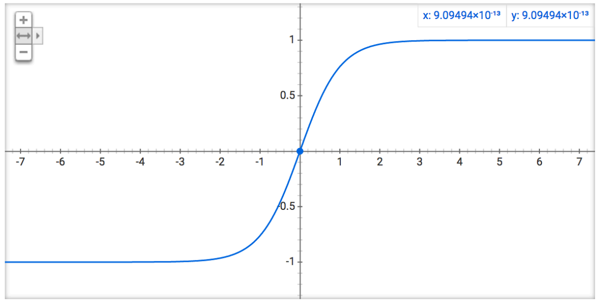
$$f_6(x') = (\max_{i \neq t}(Z(x')_i) - Z(x')_t)^+$$

- Handle box constraint by changing variables.

$$\delta_i = \frac{1}{2}(\tanh(w_i) + 1) - x_i$$

  - Since $-1 \leq \tanh(w_i) \leq 1$, we have $0 \leq x_i + \delta_i \leq 1$

Graph for (exp(x)-exp(-x))/(exp(x)+exp(-x))



- More options on optimizers: Adam.

Carlini's L^2 Adversary

- Final form: $$\text{minimize} \quad \|\frac{1}{2}(\tanh(w)+1) - x\|_2^2 + c \cdot f(\frac{1}{2}(\tanh(w)+1)$$
  $$f(x') = \max(\max\{Z(x')_i : i \neq t\} - Z(x')_t, -\kappa)$$
- How to choose c?
  - Too large, always gets $f(x^*) \leq 0$, but the L2 distance might be large.
  - Too small, may not get $f(x^*) \leq 0$, attack fails.
  - Binary search!
- Another trick: Multiple starting-point gradient descent.

Carlini's L^0 Adversary

- Find out unimportant pixels and fix the values, iteratively.
- Iterative algorithm:
    1. Run L2 adversary on x' and restore the fixed pixels, terminate if attack fails.
    2. Compute $g = \nabla f(x + \delta)$
    3. Select pixel $i = \arg\min_i g_i \cdot \delta_i$ and fix it. Goto 1.
- How to select c for L2?
    - Search from a very low value until L2 is successful. Double c till threshold.
- Warm-start trick.
- Compared with JSMA
    - Grows Vs. Shrinks an allowed set; Less like salt and pepper perturbations.

Carlini's L^{∞} Adversary

- Naïve form

$$\text{minimize} \quad c \cdot f(x + \delta) + \|\delta\|_{\infty}$$

  - Only penalize the (single) largest entry, easy to get oscillating.
- Revised form

$$\text{minimize} \quad c \cdot f(x + \delta) + \cdot \sum_{i} \left[(\delta_i - \tau)^{+} \right]$$

  - Reduce tau (x0.9) iteratively if all entries smaller than tau.
  - Choose c: the same as L0
  - Warm-start iteration.

Experimental Results

- Produce adversarial examples with smaller L^p
  - Logits-based objective function instead of loss
  - Handle box constraint by using tanh()
  - Tricks: warm-start search, multi starting points.
  - …
- Effective on Defensive distillation.
  - Bypassing softmax().
- Significantly slower (not suited for adversarial training)
  - L^0: 2x – 10x slower than optimized JSMA
  - L^2 and L^∞ : 10x – 100x slower.

Conclusion

- Improved L^2, L^∞ and L^0 attack methods.
- Proved defensive distillation is not a good defense.
- Towards evaluation of robustness.