On the Importance of Single Directions for Generalization

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https://qdata.github.io/deep2Read/
Introduction

Basic Premise and Motivation

- Research has suggested that DNNs can memorize entire datasets such as ImageNet
- Despite this, DNNs still generalize well; why do some networks generalize better than others?
- Other work: flatness of minima and PAC-bayes bounds, information content stored in network weights, SGD encourages generalization,…
- Focus on ablation analyses to measure reliance of network on single directions
- Define single direction in activation space as activation of single unit or feature map or some linear comb. of units in response to some input
Approach
Models and Datasets

- Three models: 2-hidden layer MLP trained on MNIST, 11-layer CNN trained on CIFAR-10, 50-layer residual network trained on ImageNet
- ReLU non-linearities applied to all layers but output
- Batch normalization was used for all networks
- Partially Corrupted Labels: used datasets with differing fractions of randomized labels to control degree of memorization: distribution of labels maintained, but any patterns were broken
Approach

Perturbation Analyses: Ablations

- Measured importance of single direction to a network by seeing how performance degrades once direction influence was removed
- To remove coordinate-aligned single direction, clamped activity in the direction to fixed value
- Ablations were performed on single units in MLPs or entire feature maps in CNNs and performed in activation space, not weight space
- See how network performance degrades as increasing subsets of single directions are ablated
- Decided to clamp to 0
Approach

Perturbation Analyses: Noise

- To test network dependence on random single directions (as opposed to coordinate-aligned), add Gaussian noise to all units with zero mean and progressively increasing variance.
- Normalize variance by empirical variance of unit’s activations across training set.
Approach

Quantifying Class Selectivity of Individual Units

- Used metric inspired by selectivity indices used in systems neuroscience

\[
selectivity = \frac{\mu_{\text{max}} - \mu_{-\text{max}}}{\mu_{\text{max}} + \mu_{-\text{max}}}
\]

where \( \mu_{\text{max}} \) is highest class-conditional mean activity and \( \mu_{-\text{max}} \) is mean activity for all other classes

- Metric ranges from 0 (unit’s average activity same for all classes) to 1 (unit only active for inputs of single class)
Approach
Quantifying Class Selectivity of Individual Units

- Imperfect measure of selectivity: unit with little information about every class would have low index, but would measure discriminability of classes
- Replicate results using mutual information which highlights units with information about multiple classes
Experiments
Intuition

▶ Consider two large networks: one which memorizes the dataset, one which learns the structure and thus generalizes well
▶ Memorizing network should have larger minimal description length than generalizing network
▶ Therefore, memorizing network should use more capacity, and by extension, more single directions
Experiments

Effect of Memorization on Single Direction Dependence

- Test whether memorization leads to greater dependence on single directions: train variety of networks on datasets with differing amounts of random labels and evaluate performance as more single directions were ablated.
- More corrupted labels increased sensitivity to ablations.
Experiments
Effect of Memorization on Random Single Directions

- Repeated similar experiment with random noise perturbation; similar findings
- Graphs show MLP on MNIST (a) and CNN on CIFAR-10 (b)
Experiments

Networks Trained on Same Data

▶ Also want to see if conclusions apply to networks which are not forced to memorize set (i.e. trained with uncorrupted data)
▶ Trained 200 networks on CIFAR-10 with different initializations and training data order
▶ Compared 5 networks with best generalization and 5 networks with worst; similar findings as before
▶ Plot area under cumulative ablation curve for all 200 networks
Experiments
Single Directions as Signal for Model Selection

- Can single direction reliance be used to estimate generalization performance without need for held-out test set?
- Trained MLP on MNIST and measured area under cumulative ablation curve (AUC) over course of training
- AUC starts to drop when test and train accuracies start to diverge, AUC and test loss negatively correlated
Experiments
Single Directions as Signal for Model Selection

- Can single direction reliance be used for hyperparameter selection?
- Trained 192 CIFAR-10 models with different hyperparameters
- AUC and test accuracy highly correlated in hyperparameter sweep
Experiments
Relation to Dropout

- Similar to using dropout at training time; seems to discourage reliance on single directions
- However, network is only robust to ablations up to dropout fraction
- With enough capacity, a network can guard against dropout by making multiple copies of a single direction; however, network will only make minimum number of copies
- Network robust to dropout as long as all redundant single directions were not removed at the same time
Experiments

Relation to Dropout

- Trained MLPs on MNIST with dropout probabilities of 0.1, 0.2, and 0.3 on both corrupted and unmodified labels
- Took longer to converge and converged to worse solutions; implies that memorization is discouraged
- However, past dropout, networks much more sensitive to ablations; suggests dropout is an effective regularizer, but only until dropout fraction
Experiments
Relation to Batch Normalization

- Batch normalization does appear to discourage reliance on single directions
- Trained CNNs on CIFAR-10 with and without and measured robustness to ablation
Experiments
Class Selectivity and Importance

- Results suggest that networks less reliant on single directions generalize better
- Counter-intuitive to past work in neuroscience and deep learning which highlight important of single units/feature maps which are selective for particular features of classes
- Test whether class-selectivity of single directions affects importance of directions to a network’s output
Experiments
Class Selectivity and Importance

- Test if batch normalization influences distribution of information about class across single directions
- Use selectivity index from before, trained 4 uncorrupted models on CIFAR-10
- Batch normalization actually discourages presence of feature maps with concentrated class information; raises question of whether highly selective feature maps are beneficial
Experiments
Class Selectivity and Importance

- Determine if selectivity of unit affects impact of ablating said unit
- For MLPs trained on MNIST, very small correlation (Spearman’s: 0.095)
- Many highly-selective units had minimal impact when ablated
- Similar results for CNNs on CIFAR-10
- Actually, CIFAR-10 had negative correlation; found to be driven by early network layers
- In all 3 networks, earlier ablations more impactful
- Repeated with mutual information and got similar results
- Overall: selective and non-selective units are similarly important
Experiments
Class Selectivity and Importance
Experiments
Selectivity and L1-norm

- Compared class selectivity to L1-norm of filter weights, a metric which is a good predictor of feature map importance
- Found to be largely unrelated (if not negatively related)
- Suggests class selectivity may in fact be detrimental to network performance; more research needs to be done
Related Work

- Direct inspiration from Zhang et al. (2017); replicated results using partially corrupted labels and answer the posed question: is there an empirical difference between networks which memorize and those which generalize?
- Linking generalization to sharpness of minima
- Contextualizing generalization in information theory
- Analysis on properties of models trained on corrupted labels
- Perturbation analyses: model pruning, finding maximally important direction, highlighting single selective units, ...
- Concept selectivity metric
Discussion and Future Work

Conclusion

- Taken an empirical approach to comparing memorizing and generalizing networks
- Found the generalizing ability is related to reliance on single directions in models trained on both corrupted and uncorrupted data, and also over the course of training for a single network
- Showed that batch normalization discourages dependence on single directions
- Class selectivity largely uncorrelated to importance of unit to output; batch normalization actually decreases selectivity, which suggests that class selectivity may harm output
Discussion and Future Work

Future Work

- Construct regularizer which penalizes dependence on single directions
- Could assess generalization performance without sacrificing training data to be used as validation set
- Could use single direction reliance as a signal for early-stopping or hyperparameter searching
- Find extent to which train and test set overlap affects single direction dependence
References